H2
Stability of a Floating Body

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Contents

Section			Page
1	**INTRODUCTION**		1
	1.1	Description of Apparatus	1
	1.2	Theory of Stability of a Floating Body	2
	1.3	Installation Instructions	4
	1.4	Routine Care and Maintenance	4
	1.5	Magnets	4
2	**EXPERIMENTAL PROCEDURE**		5
3	**RESULTS AND CALCULATIONS**		7
4	**ALTERNATIVE THEORY AND PROCEDURE**		9
	4.1	Introduction	9
	4.2	Experimental Determination of Stability	10
	4.3	Alternative Experimental Procedure	12

SECTION 1.0 INTRODUCTION

The question of the stability of a body, such as a ship, which floats in the surface of a liquid, is one of obvious importance. Whether the equilibrium is stable, neutral or unstable is determined by the height of its gravity, and in this experiment the stability of a pontoon may be determined with its centre of gravity at various heights. A comparison with calculated stability may also be made.

1.1 Description of Apparatus

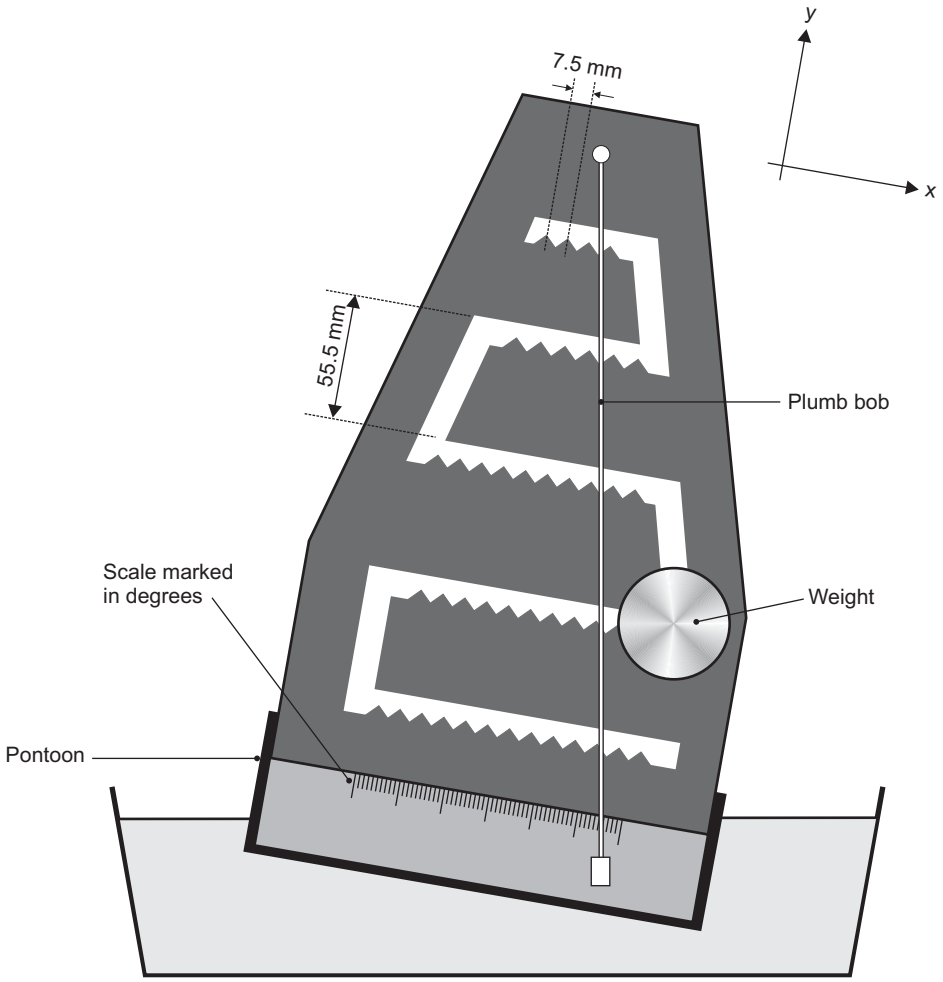
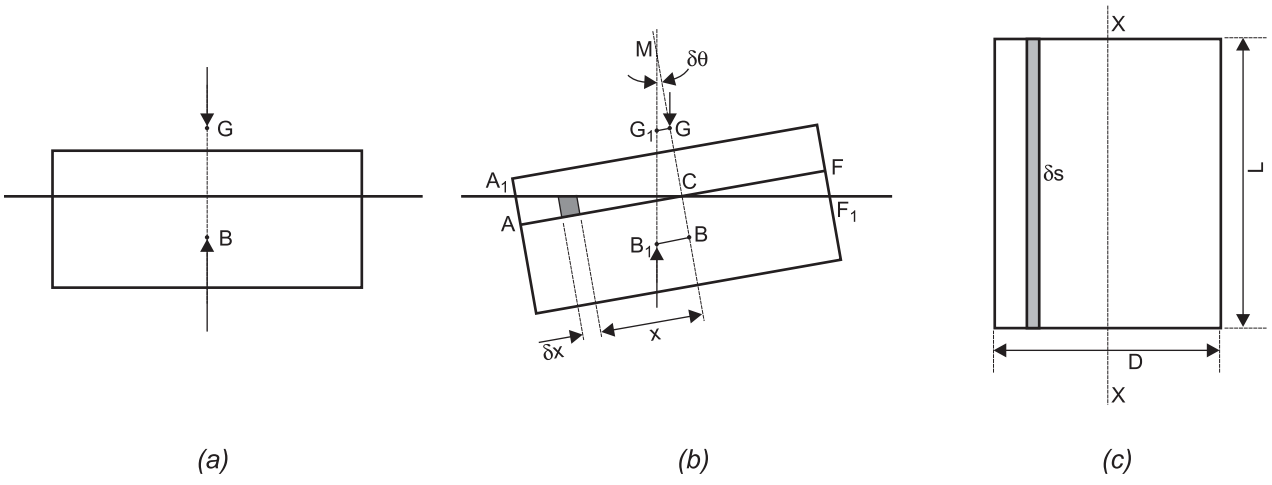


Figure 1 Arrangement of floating pontoon

The arrangement of the apparatus is shown in Figure 1. A pontoon of rectangular form floats in water and carries a plastic sail, with five rows of V-slots at equi-spaced heights on the sail. The slots' centres are spaced at 7.5 mm intervals, equally disposed about the sail centre line. An adjustable 'jockey' weight, consisting of two machined cylinders which can be screwed together, fits into the V-slots on the sail; this can be used to change the height of the centre of gravity and the angle of list of the pontoon. A plumb bob is suspended from the top centre of the sail and is used in conjunction with the scale fitted below the base of the sail to measure the angle of list.

1.2 Theory of Stability of a Floating Body



(a) (b) (c)

Figure 2 Derivation of Stability of Floating Pontoon

Consider the rectangular pontoon shown floating in equilibrium on an even keel, as shown in the cross section of Figure 2(a). The weight of the floating body acts vertically downwards through its centre of gravity G and this is balanced by an equal and opposite buoyancy force acting upwards through the centre of buoyancy B, which lies at the centre of gravity of the liquid displaced by the pontoon.

To investigate the stability of the system, consider a small angular displacement $\delta\theta$ from the equilibrium position as shown on Figure 2(b). The centre of gravity of the liquid displaced by the pontoon shifts from B to B_1. The vertical line of action of the buoyanct force is shown on the figure and intersects the extension of line BG at M, the metacentre.

The equal and opposite forces through G and B_1 exert a couple on the pontoon, and provided that M lies above G (as shown in Figure 2(b)) this couple acts in the sense of restoring the pontoon to even keel, i.e. the pontoon is stable. If, however, the metacentre M lies below the centre of gravity G, the sense of the couple is to increase the angular displacement and the pontoon is unstable. The special case of neutral stability occurs when M and G coincide.

Figure 2(b) shows clearly how the metacentric height GM may be established experimentally using the adjustable weight (of mass ω) to displace the centre of gravity sideways from G. Suppose the adjustable weight is moved a distance δx from its central position. If the weight of the whole floating assembly is W, then the corresponding movement of the centre of gravity of the whole in a direction parallel to the base of the pontoon is $\frac{\omega}{W}\delta x_1$. If this movement produces a new equilibrium position at an angle of list $\delta\theta$, then in Figure 2(b), G_1 is the new position of the centre of gravity of the whole, i.e.

$$GG_1 = \frac{\omega}{W}\delta x_1 \qquad (1)$$

Now, from the geometry of the figure:

$$GG_1 = GM \cdot \delta\theta \qquad (2)$$

Eliminating GG_1 between these equations we derive:

$$GM = \frac{\omega}{W} \cdot \frac{\delta x_1}{\delta\theta} \qquad (3)$$

or in the limit:

$$GM = \frac{\omega}{W}\left(\frac{dx_1}{d\theta}\right) \tag{4}$$

The matacentric height may thus be determined by measuring $\left(\frac{dx_1}{d\theta}\right)$ knowing ω and W. Quite apart from experimental determinations, BM may be calculated from the mensuration of the pontoon and the volume of liquid which it displaces. Referring again to Figure 2(b), it may be noted that the restoring moment about B, due to shift of the centre of buoyancy to B_1, is produced by additional buoyancy represented by triangle AA_1C to one side of the centre line, and reduced buoyancy represented by triangle FF_1C to the other. The element shaded in Figure 2(b) and (c) has and area δs in plan view and a height xδθ in vertical section, so that its volume is xδsδθ. The weight of liquid displaced by this element is wxδsδθ, where w is the specific weight of the liquid, and this is the additional buoyancy due to the element. The moment of this elementary buoyancy force about B is $wx^2\delta s\delta\theta$, so that the total restoring moment about B is given by the expression:

$$w\delta\theta\int x^2 ds$$

where the integral extends over the whole area s of the pontoon at the plane of the water surface. The integral may be referred to as I, where:

$$I = \int x^2 ds \tag{5}$$

the second moment of area of s about the axis XX.

The total restoring moment about B may also be written as the total buoyancy force, wV, in which V is the volume of liquid displaced by the pontoon, multiplied by the lever arm BB_1. Equating this product to the expression for total restoring moment derived above:

$$wV \cdot BB_1 = w\delta\theta\int x^2 ds$$

Substituting from Equation 5 for the integral and using the expression:

$$BB_1 = BM \cdot \delta\theta \tag{6}$$

which follows from the geometry of Figure 2(b), leads to:

$$BM = \frac{I}{V} \tag{7}$$

This result, which depends only on the mensuration of the pontoon and the volume of liquid which it displaces, will be used to check the accuracy of the experiment. It applies to a floating body of any shape, provided that I is taken about an axis through the centroid of the area of the body at the plane of the water surface, the axis being perpendicular to the place in which angular displacement takes place. For a rectangular pontoon, B lies at a depth below the water surface equal to half the total depth of immersion, and I may readily be evaluated in terms of the dimensions of the pontoon as:

$$I = \int x^2 ds = \int_{-D/2}^{D/2} x^2 L dx = \frac{1}{12} L D^3 \tag{8}$$

1.3 Installation Instructions

Fit the sail into its housing on the pontoon and tighten the clamp screws. Check that the plumb bob hangs vertically downwards on its cord and is free to swing across the lower scale.

1.4 Routine Care and Maintenance

After use, the water in the tank should be poured away and the pontoon and tank wiped dry with a lint-free cloth. The pontoon should never be left permanently floating in the water.

1.5 Magnets

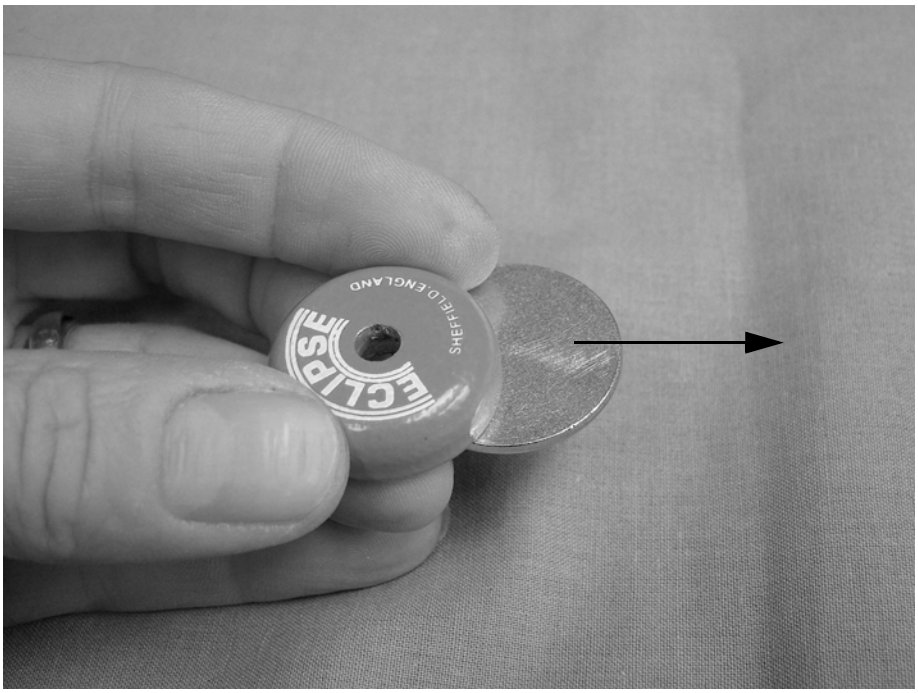


Figure 3 Remove Keep

TecQuipment supply two small magnets with the equipment to help 'trim' the balance of the pontoon. These magnets have a metal 'keep' that helps keep their magnetism when they are not being used or when packed for transport. Remove the keeps before you use the magnets or they will not stick to the metal parts on the equipment. Replace the keeps when you have finished with the magnets or need to repack them.



This equipment uses powerful magnets - always put the protective metal plates ('keeps') back on the magnets when they are not in use. This helps to contain and keep the magnetism strong for several years.

Keep any sensitive mechanical watches or instruments away from the magnets.

Always slide the magnets onto the metal surface of the pontoon. Never allow them to impact against it, as you may trap the skin of your fingers or damage the paintwork.

SECTION 2.0 EXPERIMENTAL PROCEDURE

The total mass of the apparatus (including the two magnetic weights, but not the jockey weight) is written on a label affixed to the sail housing.

The height of the centre of gravity may be found as follows (refer to Figure 4):

(i) Fit the two magnetic weights to the base of the pontoon.

(ii) Fit the hook of the centre of gravity cord through the hole in the side of the sail, ensuring that the small weight is free to hang down on the side of the sail which has the scored centre line.

(iii) Clamp the adjustable weight into the V-slot on the centre line of the lowest row and suspend the pontoon from the free end of the thick cord. Mark the point where the plumb line crosses the sail centre line with typists' correcting fluid or a similar marking fluid.

(iv) Repeat paragraph (iii) for the other four rows.

With the adjustable weight situated in the centre of one of the rows, allow the pontoon to float in water and position the two magnetic weights on the base of the pontoon to trim the vessel. When the vessel has been trimmed correctly, the adjustable weight may be moved to positions either side of the centre line for each of the five rows. At each position the displacement can be determined by the angle the plumb line from the top of the sail makes with the scale on the sail housing.

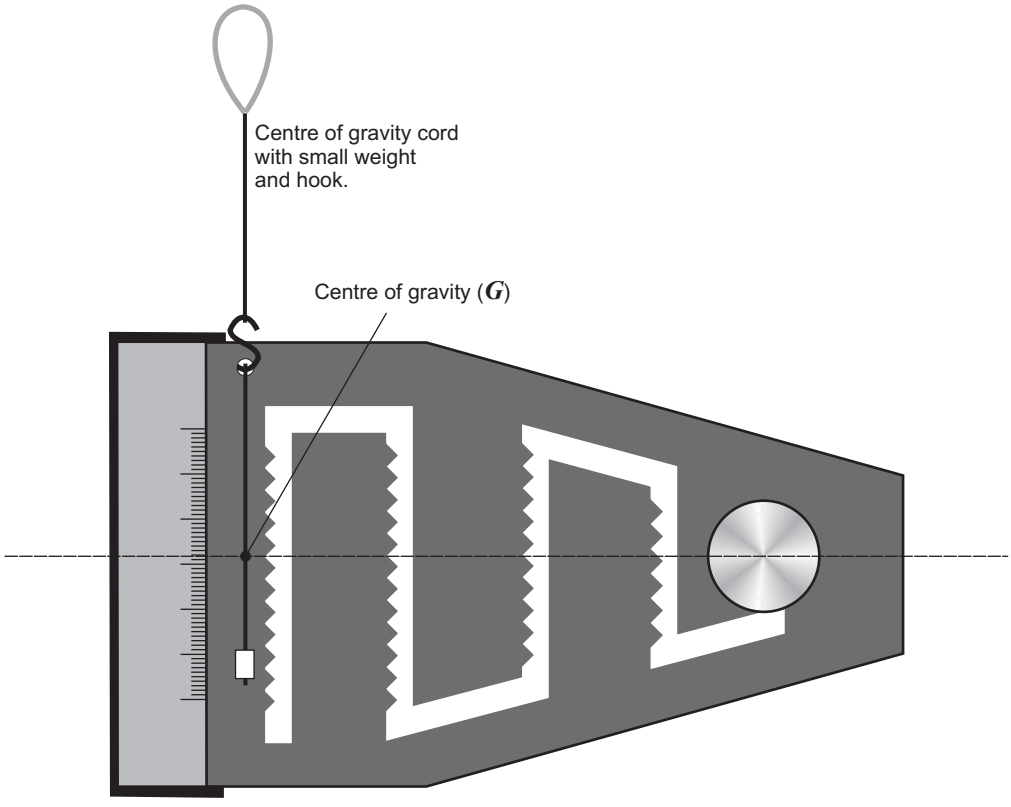


Figure 4 Method of finding centre of gravity

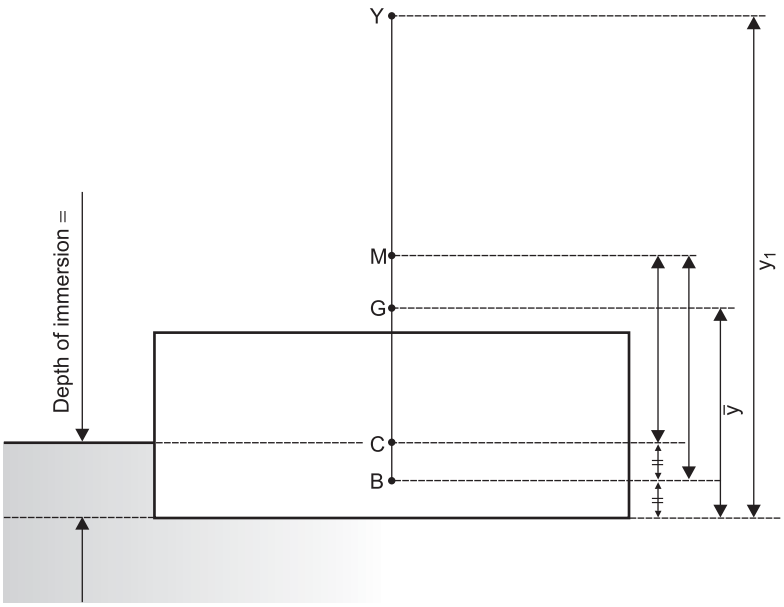


Figure 5 Standard dimensions of pontoon

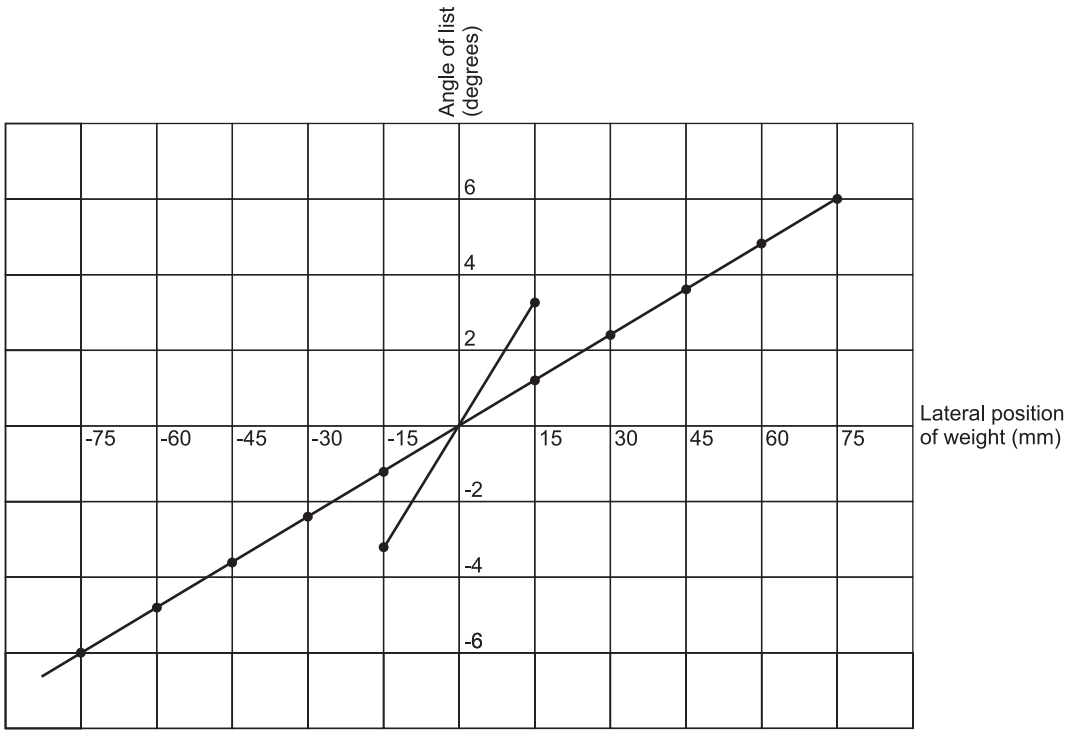


Figure 6 Variations of angle of list with lateral position of weight (heights shown are maximum and minimum)

SECTION 3.0 RESULTS AND CALCULATIONS

Any results are typical only. Actual results may differ slightly, determined by slight differences in manufacturing tolerances and materials.

Total weight of floating assembly (W)	=	kg
Adjustable weight (ω)	=	kg
Breadth of pontoon (D)	=	m
Length of pontoon (L)	=	m
Second moment of area $I = \frac{LD^3}{12}$	=	m^4
Volume of water displaced $V = \frac{W}{10^3}\rho$	=	m^3
Height of metacentre above centre of buoyancy $BM = \frac{I}{V}$	=	m
Depth of immersion of pontoon $= \frac{V}{LD}$	=	m
Depth of centre of buoyancy $CB = \frac{V}{2LD}$	=	m

It is suggested that Figure 5 is marked up to be referred to each time the apparatus is used. Note that when measuring the heights $\bar{y}$ and y_1, as it is only convenient to measure from the inside floor of the pontoon, the thickness of the sheet metal bottom should be added to $\bar{y}$ and y_1 measurements. The position of G (and hence the value the value of $\bar{y}$) and a corresponding value of y was marked earlier in the experiment when the assembly was balanced.

The height of $\bar{y}$ and G above the base will vary with the height y of the adjustable weight above the base, according to the equation:

$$\bar{y} = y_1\frac{\omega}{W} + A \qquad (9)$$

where A is a constant which pertains to the centre of gravity of the pontoon and the height of the adjustable weight.

Using one set of results for the centre of gravity of the pontoon and the height of the adjustable weight, then $\bar{y}$ and y_1 can be measured and the constant A calculated. This can then be used in calculations for subsequent heights of $\bar{y}$ and y_1 which can be checked against the markings made earlier.

Values of angles of list produced by lateral movement of the adjustable weight height y_1 should be recorded in the form of Table 1. A graph (Figure 6) for each height y_1, of lateral position of adjustable weight against angles of list, can then be plotted.

Note: Decide which side of the sail centre line is to be termed negative and then term list angles on that side negative.

Height of jockey weight y_1 mm (i)	Angles of list for adjustable weight lateral displacement from sail centre line x_1 mm (ii)														
	-52.5	-45	-37.5	-30	-22.5	-15	-7.5	0	7.5	15	22.5	30	37.5	45	52.5

Table 1 Values of list angles for height and position of adjustable weight

From Figure 6, for the five values of y and the corresponding values of $dx_1/d\theta$ can be extracted. Using Equation 4 values of GM can be obtained. Using Equation 9 and knowing the immersion depth, values of CG can be derived. Also, since CM = CG + GM, values of CM can be calculated. The above values should be calculated and arranged in tabular form as show in Table 2.

Height of adjustable weight y_1 mm (i)	Height of G above water surface CG (mm) (ii)	$\frac{dx_1}{d\theta}$ (mm/°) (iii)	Metacentric height GM (mm) (iv)	Height of M above water surface CM (mm) (v)

Table 2 Derivation of metacentric height from experimental results

The values of $dx_1/d\theta$ can now be plotted against CG, the height of G above the water line. Extrapolation of this plot will indicate the limiting value of CG above which the pontoon will be unstable.

SECTION 4.0 ALTERNATIVE THEORY AND PROCEDURE

This section gives alternative and equally valid theory and experiment procedure.

Any results are typical only. Actual results may differ slightly, determined by slight differences in manufacturing tolerances and materials.

4.1 Introduction

When designing a vessel such as a ship, which is to float on water, it is clearly necessary to be able to establish beforehand that it will float upright in stable equilibrium.

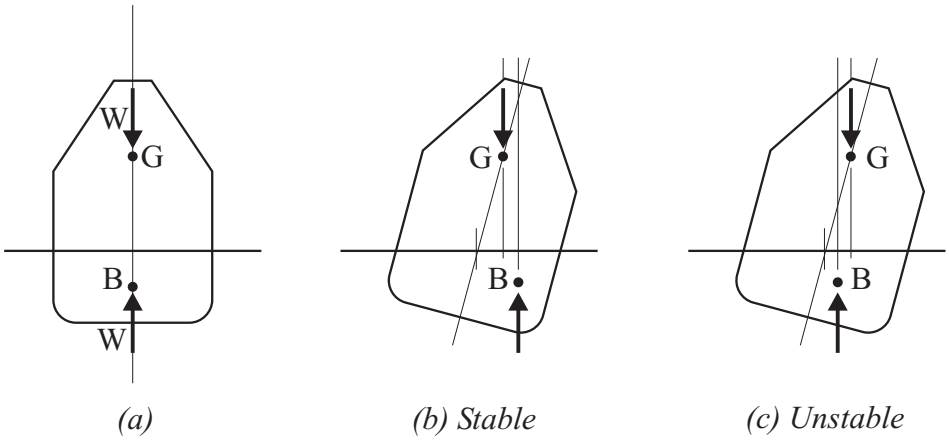


Figure 7 Forces Acting on a Floating Body

Figure 7 (a) shows such a floating body, which is in equilibrium under the action of two equal and opposite forces, namely, its weight W acting vertically downwards through its centre of gravity G, and the buoyancy force, of equal magnitude W, acting vertically upwards at the centre of buoyancy B. This centre of buoyancy is located at the centre of gravity of the fluid displaced by the vessel. When in equilibrium, the points G and B lie in the same vertical line. At first sight, it may appear that the condition for stable equilibrium would be that G should lie below B. However, this is not so.

To establish the true condition for stability, consider a small angular displacement from the equilibrium position, as shown in Figures 7(b) and 7(c). As the vessel tilts, the centre of buoyancy moves sideways, remaining always at the centre of gravity of the displaced liquid. If, as shown on Figure 7(b), **the weight and the buoyancy forces together produce a couple which acts to restore the vessel to its initial position, the equilibrium is stable. If however, the couple acts to move the vessel even further from its initial position, as in Figure 7(c), then the equilibrium is unstable**. The special case when the resulting couple is zero represents the condition of neutral stability. It will be seen from Figure 7(b) that it is perfectly possible to obtain stable equilibrium when the centre of gravity G is located above the centre of buoyancy B.

In the following text, we shall show how the stability may be investigated experimentally, and then how a theoretical calculation can be used to predict the results.

4.2 Experimental Determination of Stability

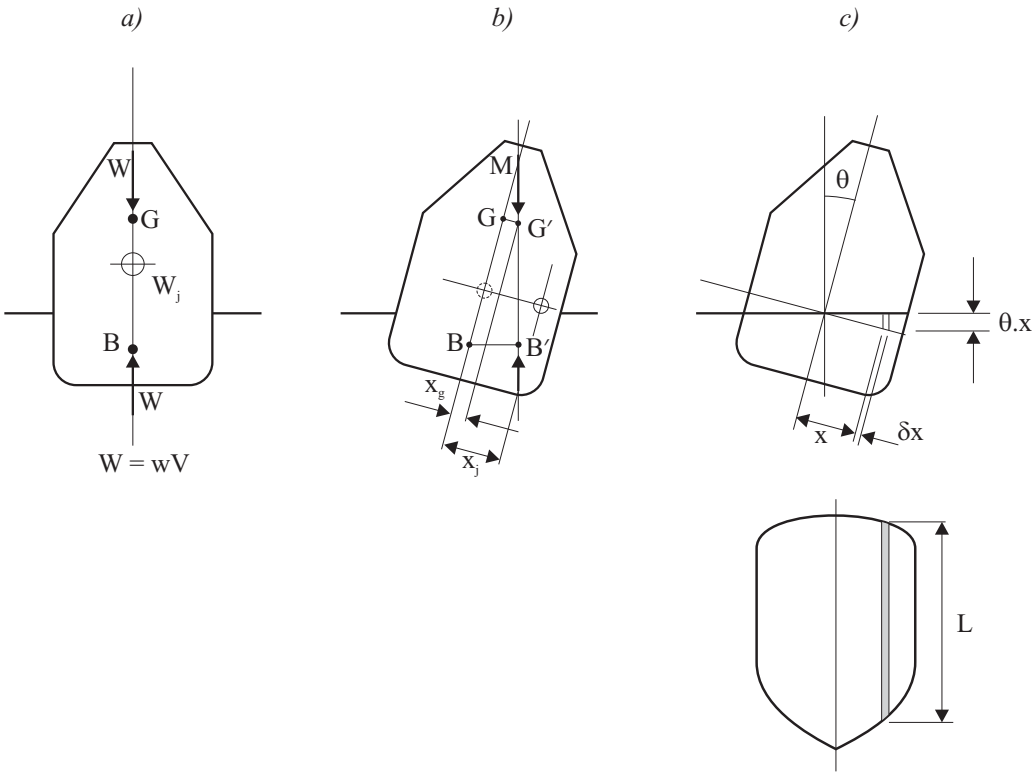


Figure 8 Derivation of conditions for stability

Figure 8 (a) shows a body of total weight W floating on even keel. The centre of gravity G may be shifted sideways by moving a jockey of weight W_j across the width of the body. When the jockey is moved a distance x_j, as shown in Figure 8(b), the centre of gravity of the whole assembly moves to G'. The distance GG', denoted by x_g, is given from elementary statics as

$$x_g = \frac{W_j x_j}{W} \qquad (10)$$

The shift of the centre of gravity causes the body to tilt to a new equilibrium position, at a small angle θ to the vertical, as shown in Figure 8(b), with an associated movement of the centre of buoyancy from B to B'. The point B' must lie vertically below G', since the body is in equilibrium in the tilted position. Let the vertical line of the upthrust through B' intersect the original line of upthrust BG at the point M, called the **metacentre**. We may now regard the jockey movement as having caused the floating body to swing about the point M. Accordingly, **the equilibrium is stable if the metacentre lies above G**. Provided that θ is small, the distance GM is given by

$$GM = \frac{x_g}{\theta}$$

where θ is in circular measure. Substituting for x_g from Equation 10 gives the result

$$GM = \frac{W_j}{W} \cdot \frac{x_j}{\theta} \qquad (11)$$

The dimension GM is called the metacentric height. In the experiment described below, it is measured directly from the slope of a graph of x_j against θ, obtained by moving a jockey across a pontoon.

Analytical Determination of BM

A quite separate theoretical calculation of the position of the metacentre can be made as follows.

The movement of the centre of buoyancy to B' produces a moment of the buoyancy force about the original centre of buoyancy B. To establish the magnitude of this moment, first consider the element of moment exerted by a small element of **change** in displaced volume, as indicated on Figure 8(c). An element of width x, lying at distance x from B, has an **additional** depth θ.x due to the tilt of the body. Its length, as shown in the plan view on Figure 8(c), is L. So the volume δV of the element is:

$$\delta V = \theta \cdot x \cdot L \cdot \delta x = \theta L x \delta x$$

and the element of **additional** buoyancy force δF is

$$\delta F = w \cdot \delta V = w\theta L x \delta x$$

where w is the specific weight of water. The element of moment about B produced by the element of force is δM, where

$$\delta M = \delta F x = w\theta L x^2 \delta x$$

The total moment about B is obtained by integration over the whole of the plan area of the body, in the plane of the water surface:

$$M = w\theta \int L x^2 dx = w\theta I \qquad (12)$$

In this, 'I' represents the second moment, about the axis of symmetry, of the water plane area of the body.

Now this moment represents the movement of the upthrust wV from B to B', namely, wV.BB'. Equating this to the expression for M in Equation 12:

$$wV \cdot BB' = w\theta I$$

From the geometry of the figure, we see that

$$BB' = \theta \cdot BM$$

and eliminating BB' between these last two equations gives BM as

$$BM = \frac{I}{V} \qquad (13)$$

For the particular case of a body with a rectangular planform of width D and length L, the second moment I is readily found as:

$$I = \int_{-D/2}^{D/2} L x^2 dx = L \int_{-D/2}^{D/2} x^2 dx = L\left[\frac{x^3}{3}\right]_{-D/2}^{D/2} = \frac{LD^3}{12} \qquad (14)$$

Now the distance BG may be found from the computed or measured positions of B and of G, so the metacentric height GM follows from Equation 13 and the geometrical relationship:

$$GM = BM - BG \qquad (15)$$

This gives an independent check on the result obtained experimentally by traversing a jockey weight across the floating body.

4.3 Alternative Experimental Procedure

With the jockey weight on the line of symmetry, small magnetic weights are used to trim the assembly to even keel, indicated by a zero reading on the angular scale. The jockey is then moved in steps across the width of the pontoon, the corresponding angle of tilt (over a range which is typically $\pm 8°$) being recorded at each step. This procedure is then repeated with the jockey traversed at a number of different heights.

Results and Calculations - Weight and Dimensions

Weight of pontoon (excluding jockey weight) W_p = 2.184 kgf

Weight of jockey W_j = 0.391 kgf

Total weight of floating assembly $W = W_p + W_j$ = 2.575 kgf

Pontoon displacement $V = \frac{W}{w} = \frac{2.575}{1000}$ = 2.575×10^{-3} m^3

Breadth of pontoon D = 201.8 mm = 0.2018 m

Length of pontoon L = 360.1 mm = 0.3601 m

Area of pontoon in plane of water surface
$A = LD = 0.3601 \times 0.2018$ = 7.267×10^{-2} m^2

Second Moment of Area $I = \frac{LD^3}{12} = \frac{0.3601 \times 0.20183}{12}$ = 2.466×10^{-4} m^4

Depth of immersion $OC = \frac{V}{A} = \frac{2.575 \times 10^{-3}}{7.267 \times 10^{-2}}$ = 3.54×10^{-2} m = 35.4 mm

Height of centre of buoyancy B above O $OB = BC = \frac{OC}{2}$ = 17.7 mm

Results and Calculations - Height of Centre of Gravity

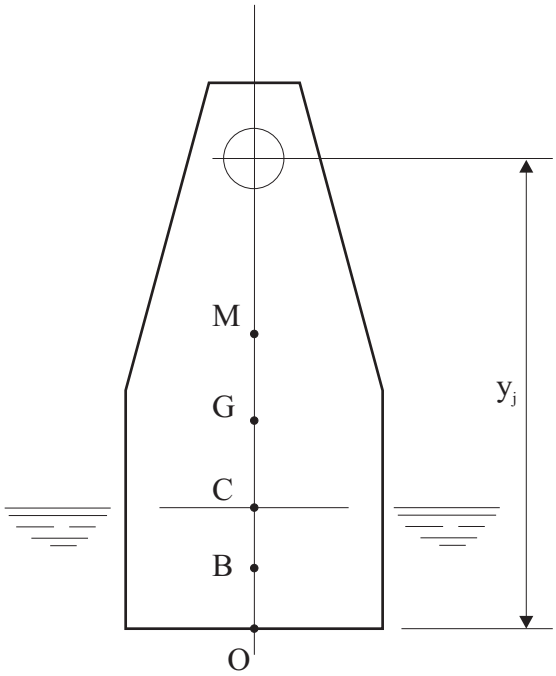


Figure 9 Height of Centre of Gravity

Figure 9 shows schematically the positions of the centre of buoyancy B, centre of gravity G, and metacentre M. O is a reference point on the external surface of the pontoon, and C is the point where the axis of symmetry intersects the plane of the water surface. The thickness of the material from which the pontoon is made is assumed to be 2 mm. The height of G above the reference point O is OG. The height of the jockey weight above O is y_j.

With the jockey weight placed in the uppermost (top) row of the sail, the following measurements were made:

Height of jockey weight above O (y_j) = 322 mm

Corresponding height of G above O (OG) = 85 mm using the gravity cord

Table 3 shows the values of OG found using the gravity cord for the 5 different heights y_j of the jockey weight.

y_j (mm)	100	155.5	211	266.5	322
OG (mm)	53	61	70	77	85

Table 3 Heights OG of G above base O of Pontoon

Experimental determination of metacentric height GM

Table 4 shows the results obtained when the pontoon was tilted by traversing the jockey weight across its width.

Jockey Height	Jockey Displacement from Centre, x_j (mm)														
y_j (mm)	-52.5	-45	-37.5	-30	-22.5	-15	-7.5	0	7.5	15	22.5	30	37.5	45	52.5
322	-	-	-	-	-	-5.5	-3.25	0.0	2	4.75	-	-	-	-	-
266.5	-	-	-	-8	-6	-4.25	-2.25	0.0	1.75	3.5	5.5	7.5	-	-	-
211	-	-	-8	-6.5	-5	-3.25	-1.75	0.0	1.5	3.25	4.75	6.25	7.75	-	-
155.5	-	-8	-6.5	-6	-4.25	-3	-1.5	0.0	1	2.5	3.75	5	6.5	7.5	-
100	-8	-6.75	-5.75	-4.5	-3.5	-2.5	-1.25	0.0	1	2	3.25	4.5	5.5	6.75	7.75

Table 4 Angles of Tilt (θ)Caused by Jockey Displacement from Centre (xj)

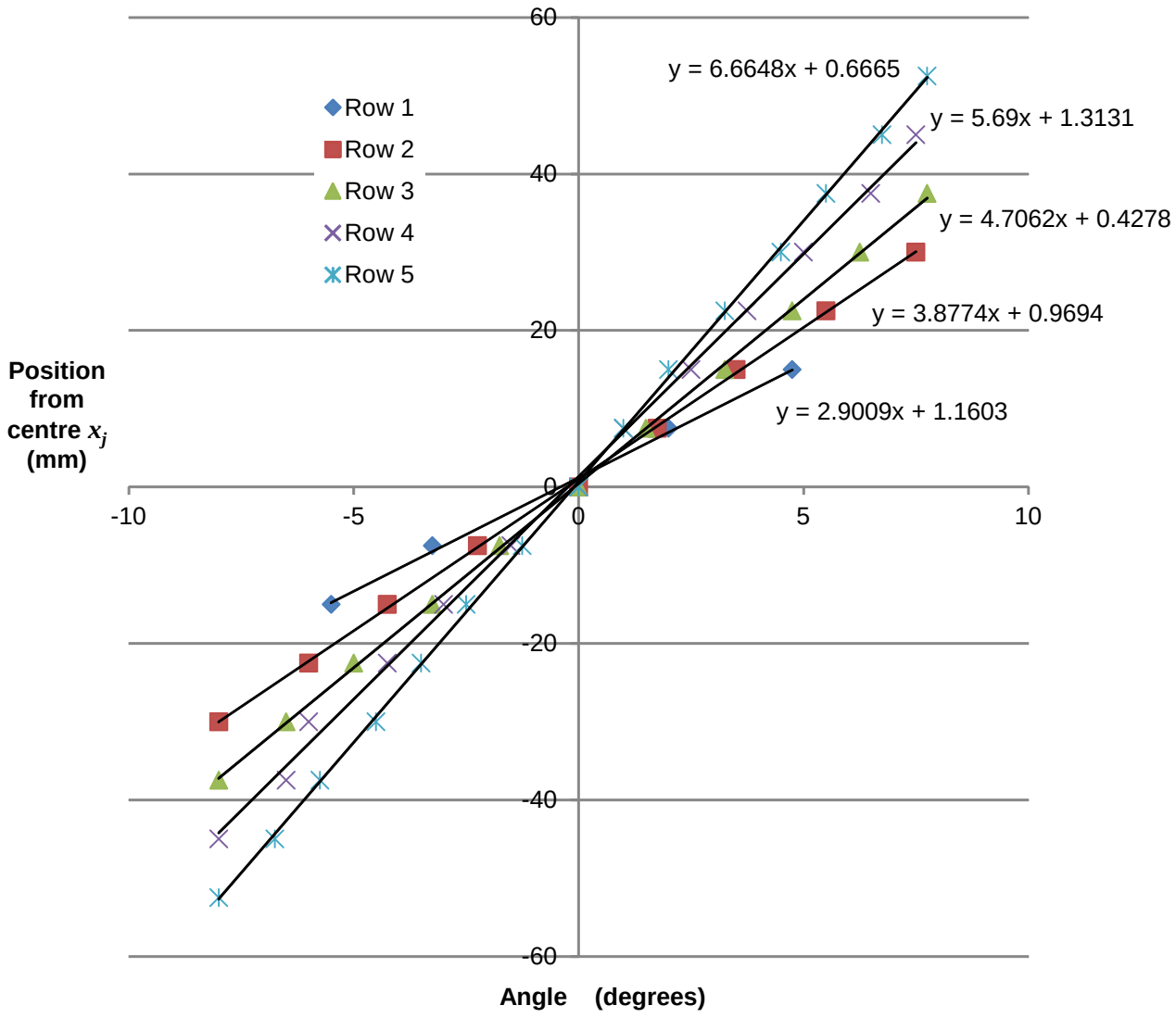


Figure 10 Variation of angle of tilt with Jockey Displacement

Figure 10 shows these results graphically. For each of the jockey heights, the angle of tilt is proportional to the jockey displacement. The metacentric height may now be found from Equation 11, using the gradients of the lines in Figure 10.

For example, for the lowest row $y_j = 100$ mm, the gradient is:

$$\frac{dx_j}{d\theta} = 6.66 \text{ mm/degree} = 6.66 \times 57.3 = 381 \text{ mm/rad}$$

Inserting this into Equation 11,

$$GM = \frac{W_j}{W} \cdot \frac{x_j}{\theta} = \frac{0.391}{2.575} \times 381 = 57.9 \text{mm}$$

This value, and corresponding values for other jockey heights, are entered into Table 5. Values of BM are also shown, derived as follows (refer to Figure 9 for notation):

$$BM = BG + GM = OG - OB + GM = OG + GM - 17.7 \text{ mm}$$

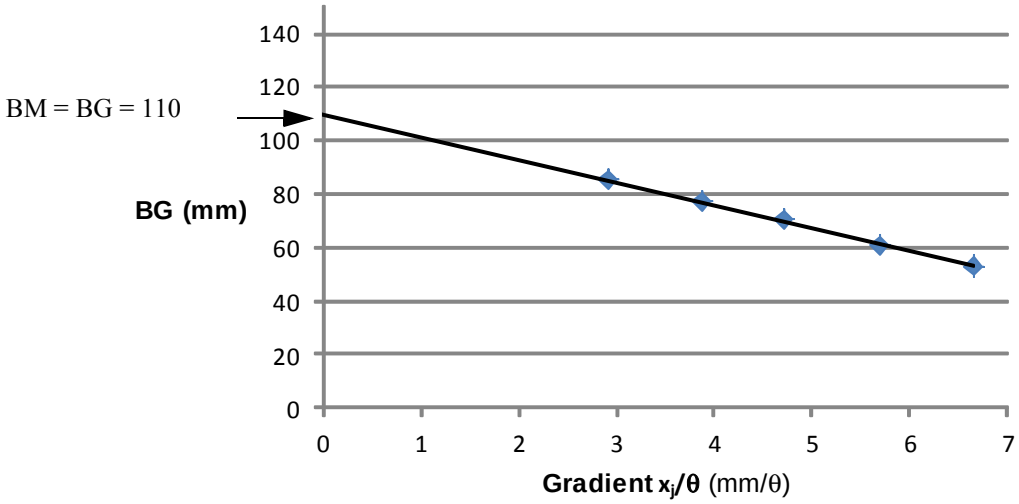


Figure 11 Variation of Stability with Metacentric Height

Jockey height (mm)	OG (mm)	x_j/θ (mm/°)	Metacentric height GM (mm)	BM (mm)
320	85	2.9	25.2	110
265	77	3.88	33.8	110
209	70	4.71	41	110
154	61	5.69	49.5	110
100	53	6.66	58	110

Table 5 Metacentric Height Derived Experimentally

As BM depends only on the mensuration and total weight of the pontoon, its value should be independent of the jockey height, and this is seen to be reasonably verified by the experimental results. The value computed from theory is

$$BM = \frac{I}{V}$$

$$\frac{2.466 \times 10^{-4}}{2.575 \times 10^{-3}} = 96 \text{ mm}$$

which is in reasonable agreement with the values obtained experimentally, considering possible measurement errors.

Another way of expressing the experimental results is presented in Figure 11, where the height BG of the centre of gravity above the centre of buoyancy is shown as a function of the slope x_j/θ. The experimental points lie on a straight line which intersects the BG axis at the value 110 mm. As BG approaches this value, $x_j/\theta \longrightarrow 0$. Namely, the pontoon may be then tilted by an infinitesimal movement of the jockey weight; it is in the condition of neutral stability. Under this condition, the centre of gravity coincides with the metacentre, viz. BM = BG. So, from Figure 11, we see that BM = 110 mm.

Discussion of Results

The experiment demonstrates how the stability of a floating body is affected by changing the height of its centre of gravity, and how the metacentric height may be established experimentally by moving the centre of gravity sideways across the body. The value established in this way agrees satisfactorily with that given by the analytical result BM = I/V.

Questions for Further Discussion

(i) What suggestions do you have for improving the apparatus?

(ii) Does the movement of the plumb-bob over the angular scale affect the results in any way? Consider, for instance, a plumb-bob of 0.005 kgf weight, displaced sideways through a distance of 90 mm as the pontoon tilts through 8°. What effect does this have, as compared with that of a corresponding displacement of the jockey weight?

(iii) What accuracy do you consider you have achieved in obtaining the analytical value of BM? If, for example, the possible uncertainty in measuring D and L is ± 2 mm, what is the corresponding uncertainty in the calculated value of BM?

(iv) How would the stability of the pontoon be affected if it were floated on a liquid with a greater density than that of water?